Unitz Home Resources
Math Vocab

Illustrative Mathematics

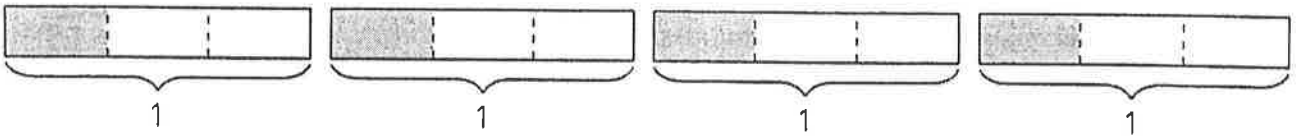
Family Support Materials

Fractions as Quotients and Fraction Multiplication

In this unit, students solve problems involving division of whole numbers with answers that are fractions (which could be in the form of mixed numbers). They develop an understanding of fractions as the division of the numerator by the denominator, that is $a \div b = \frac{a}{b}$. They then solve problems that involve the multiplication of a whole number by a fraction or mixed number.

Section A: Fractions as Quotients

In this section, students learn that fractions are quotients and can be interpreted as division of the numerator by the denominator. Students draw and analyze tape diagrams that represent sharing situations. Through the context of first sharing 1, then sharing more than 1, then sharing a number of things with increasingly more people, students notice patterns and begin to understand that in general $\frac{a}{b} = a \div b$. For example, students use the diagram below to show 4 objects being shared equally by 3 people, or $4 \div 3$, which can also be written as a fraction, $\frac{4}{3}$.



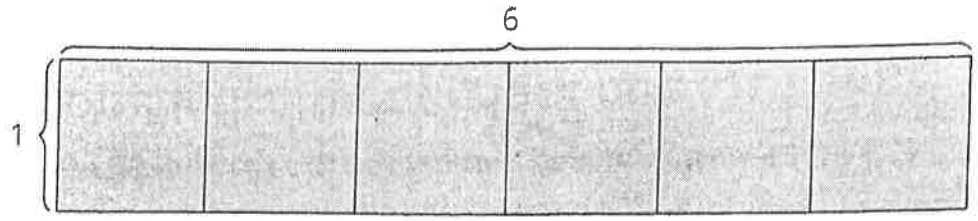
Section B: Fractions of Whole Numbers

In this section, students make connections between multiplication and division and use visual representations that can show both operations. For example, the diagram above can also represent 4 groups of $\frac{1}{3}$, or $4 \times \frac{1}{3}$. Students discover ways of finding the product of a fraction and whole number that make sense to them and connect the product to the context and diagrams. They multiply a whole number by a fraction, $\frac{a}{b} \times q$.

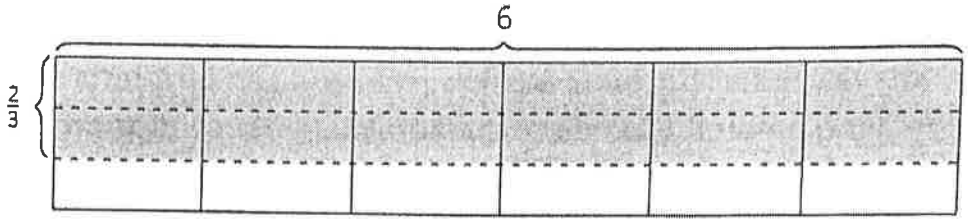
Section C: Area and Fractional Side Lengths

In this section, students use what they know about the area of rectangles with whole number side lengths to find the area of rectangles that have one whole number side length and one fractional side length.

The expression 6×1 represents the area of a rectangle that is 6 units by 1 unit.

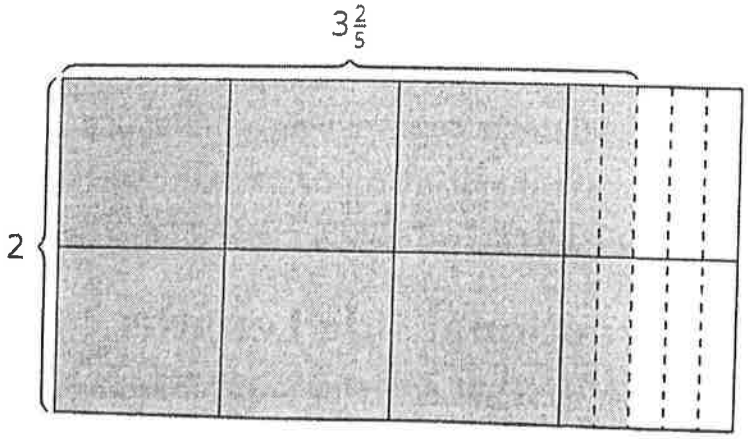


In the same way, $6 \times \frac{2}{3}$ represents the area of a rectangle that is 6 units by $\frac{2}{3}$ unit.



In addition, students see that the expressions $6 \times \frac{2}{3}$, $6 \times 2 \times \frac{1}{3}$, and $12 \times \frac{1}{3}$ can all represent the area of this same diagram.

Students analyze diagrams where one side length is a mixed number, for example a rectangle that is 2 by $3\frac{2}{5}$. They decompose the shaded region to show the whole units and the fractional units.

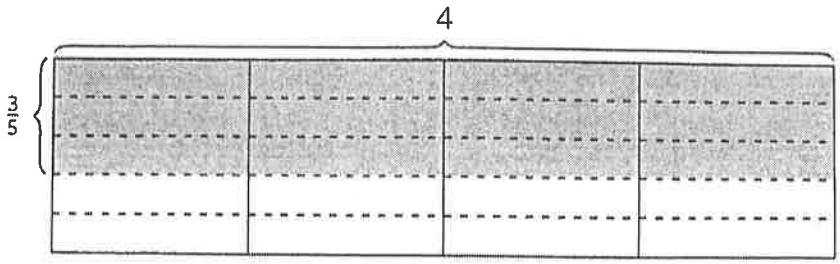


To find the area represented by this diagram, students may see two rectangles: a rectangle that is 2 units by 3 units and a rectangle that is 2 units by $\frac{2}{5}$ unit. While they may recognize that the area can be represented as $2 \times 3\frac{2}{5}$, students who see the decomposed rectangle may write $(2 \times 3) + (2 \times \frac{2}{5})$ to find the area.

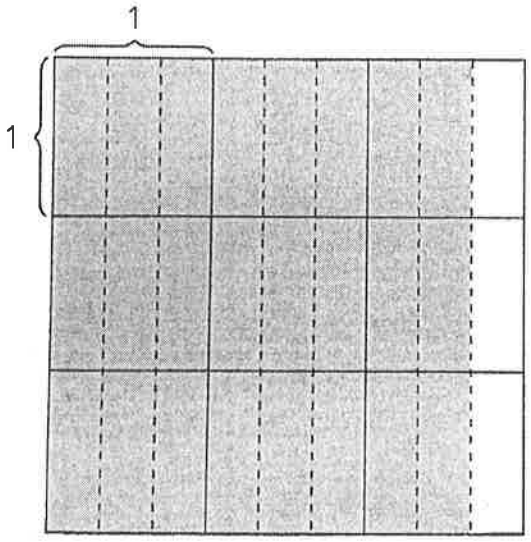
Try it at home!

Near the end of the unit, ask your student the following questions:

1. Write as many expressions as you can that represent this diagram:



2. What is the area of the following rectangle?



Questions that may be helpful as they work:

- How are the two problems similar? How are they different?
- How does your expression represent the diagram?
- How did you break up the rectangle to help you solve for the entire area?
- What are the side lengths of the rectangle?

Composite Number

A whole number with more than 1 factor pair.

6 6

1 x 6 2 x 3

6 is a composite number

Common denominator

The same denominator in two or more fractions. For instance, $\frac{1}{4}$ and $\frac{5}{4}$ have a common denominator.

$\frac{1}{3} + \frac{1}{4}$

$\frac{1}{3} \rightarrow 3, 6, 9, 12, 15$

$\frac{1}{4} \rightarrow 4, 8, 12, 16,$

$\frac{4}{12} + \frac{3}{12}$

Prime Number

A whole number that is greater than 1 and has exactly one factor pair: the number itself and 1.

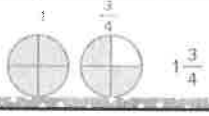
13

1 x 13

13 has only two factors - itself and 1. So it is a prime number.

Mixed Number

A mixed number is a whole number, and a proper fraction represented together.



$1\frac{3}{4}$

Factor pair of a whole number

A pair of whole numbers that multiply to result in that number. For example,5 and 4 are a factor pair of 20.

FACTOR PAIRS OF 8: 1 x 8 AND 2 x 4

Multiple of a number

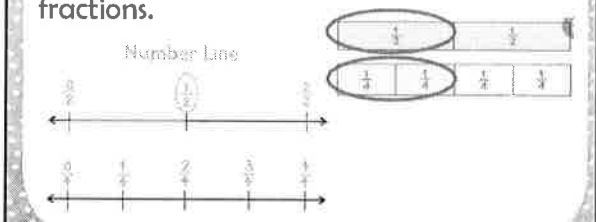
The result of multiplying that number by a whole number.For example,a multiple of 3 is 15,because 15 is a result of multiplying 3 by5.

Some multiples of 3

3 15 30 60 9

Equivalent fractions

Fractions that have the same size and describe the same point on the number line. For example, $\frac{1}{2}$ and $\frac{2}{4}$ are equivalent fractions.



denominator

The bottom part of a fraction that tells how many equal parts the whole was partitioned into.

$\frac{3}{5}$

numerator

The top part of a fraction that tells how many of the equal parts are being described.

$\frac{3}{5}$

Unit fraction

A fraction with 1 in the numerator.

$\frac{1}{4}$ $\frac{1}{2}$ $\frac{1}{23}$ $\frac{1}{12}$ $\frac{1}{8}$ $\frac{1}{5}$ $\frac{1}{6}$

quotient

The result in a division equation.

$4 = 8 \div 2$

$1 \div 2 = \frac{1}{2} = 1/2$

Divisor

the number we are dividing by which can represent the size of the groups or the number of groups.

Dividend Divisor Quotient

$\frac{3}{2} \div \frac{1}{2} = ?$

Dividend

a number to be divided by another number.

Dividend Divisor Quotient

$\frac{3}{2} \div \frac{1}{2} = ?$

Equation

A statement that includes an equal sign (=). It tells us that what is on one side of the sign is equal to what is on the other side.

0 = 0 18 / 2 = 9 36 = 4 x 9

Expression

An expression has at least 2 numbers and at least one math operation (such as addition, subtraction, multiplication and division).

3 ÷ 5 18 / 2 123 + 7

fraction

A number used to describe the parts of a whole that has been partitioned into equal parts.

one eighth

product

The result of multiplying some numbers.

6 = 2 x 3

$\frac{1}{2} \times 3 = \frac{3}{2} = 1\frac{1}{2} = 3/2$

fù hé shù

复合数

6 6

1 x 6 2 x 3

6 is a composite number

gōng fēn mǔ

公分母

$\frac{1}{3} \rightarrow 3, 6, 9, 12, 15$

$\frac{1}{4} \rightarrow 4, 8, 12, 16,$

$\frac{1}{3} + \frac{1}{4}$

$\frac{4}{12} + \frac{3}{12}$

zhì shù

质数

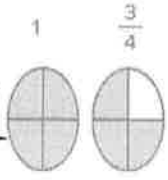
只有两个正因数（1和自己）的整数。

13

1 x 13

dài fēn shù

带分数



$1\frac{3}{4}$

yī duì yīn shù

一对因数

8 的因数：1x8 和 2x4

bèi shù

倍数

3的倍数

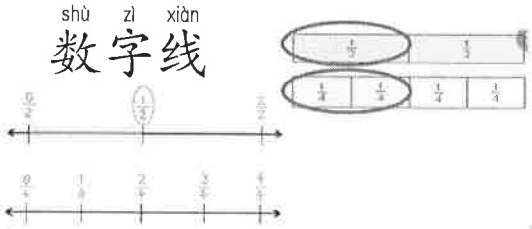
3 15 30 60 9

děng zhí fēn shù

等值分数

shù zì xiàn

数字线



fēn mǔ

分母

bǎ dān wèi

把单位 1

píng jūn fēn chéng

平均分成 $\frac{3}{5}$

ruò gān bù fēn

若干部分

fēn zǐ

分子

$\rightarrow \frac{3}{5}$

dān wèi fēn shù

单位分数

$\frac{1}{4}$ $\frac{1}{2}$ $\frac{1}{5}$ $\frac{1}{23}$ $\frac{1}{12}$ $\frac{1}{8}$ $\frac{1}{6}$

shāng

商

$\rightarrow 4 = 8 \div 2$

$1 \div 2 = \frac{1}{2} = 1/2$

chú shù

除数

$\frac{3}{2} \div \frac{1}{2} = ?$

bèi chú shù

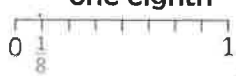
被除数

$\frac{3}{2} \div \frac{1}{2} = ?$

fēn shù

分数

八分之一 $\frac{1}{8}$



one eighth

0 $\frac{1}{8}$ 1

jī

积

$\rightarrow 6 = 2 \times 3$

$\frac{1}{2} \times 3 = \frac{3}{2} = 1\frac{1}{2} = 3/2$

děng shì

等式

yòng děng hào (=) bǎ shù zì hé suàn

用等号(=)把数字和算

shì lián jiē qǐ lái, děng hào (=)

式连接起来,等号(=)

de liǎng biān xiāng děng

的两边相等。

$0 = 0$ $18 / 2 = 9$ $36 = 4 \times 9$

suàn shì

算式

yòng jiā hào (+), jiǎn hào (−),

用加号(+),减号(−),

chéng hào (×), chú hào (÷) děng

乘号(×),除号(÷)等

fú hào lián jiē shù zì de shì zi

符号连接数字的式子。

$3 \div 5$ $18 / 2$ $123 + 7$

IM Fifth Grade Unit 2 Sample Questions

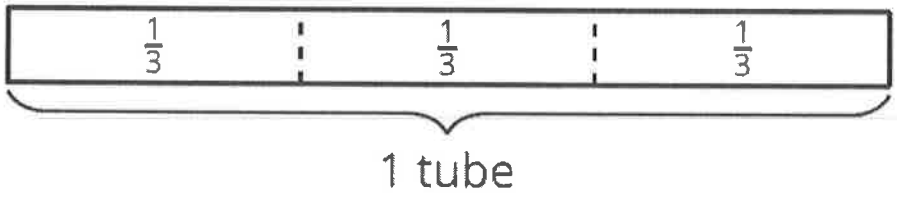
1 a. 3 students equally share 18 sheets of construction paper for an art project. How many sheets of paper does each student get? Explain or show your reasoning.

1b. 3 students equally share 1 tube of glue for an art project. How much glue does each student get? Explain or show your reasoning.

Solution :

a. 6 sheets of paper since $18 \div 3 = 6$.

b. $\frac{1}{3}$ of a tube of glue



2a. 4 hikers equally share 3 liters of water. How many liters of water does each hiker drink? Explain or show your reasoning.

2b. 4 hikers equally share 5 liters of water. How many liters of water does each hiker drink? Explain or show your reasoning.

Solution : a. Equation : $3 \div 4 = \frac{3}{4}$ liters of water.

a. $\frac{3}{4}$ liter or equivalent. Each hiker gets $\frac{1}{4}$ of each liter of water.

b. $\frac{5}{4}$ liter or equivalent. Each hiker gets $\frac{1}{4}$ of each liter of water.

b. Equation : $5 \div 4 = \frac{5}{4}$ liters of water.

IM Fifth Grade Unit 2 Sample Questions

4. Decide whether each equation is true or false. Explain or show your reasoning.

a. $3 \div 7 = \frac{3}{7}$.

b. $18 \div 5 = \frac{5}{18}$.

c. $15 \div 6 = 2\frac{1}{2}$.

Solution :

a. True. Sample response: I can divide the 3 wholes into 7 equal parts by taking $\frac{1}{7}$ of each whole and putting those parts together. That's the same as $\frac{3}{7}$ of one whole.

b. False. Sample response: This is not true since $\frac{5}{18}$ is less than 1 while $18 \div 5$ is greater than 1.

c. True. Sample response: I can first take 12 and make 6 equal portions of 2 and then there are 3 left which makes 6 equal portions of $\frac{1}{2}$.

IM Fifth Grade Unit 2 Sample Questions

5. Han cuts a 15-foot piece of rope into 4 equal parts. Decide whether each expression represents the length of each part of the rope in feet. Explain or show your reasoning.

a. $15 \div 4$

b. 4×15

c. $3\frac{3}{4}$

Solution :

a. Yes, this is 1 of four equal groups making 15.

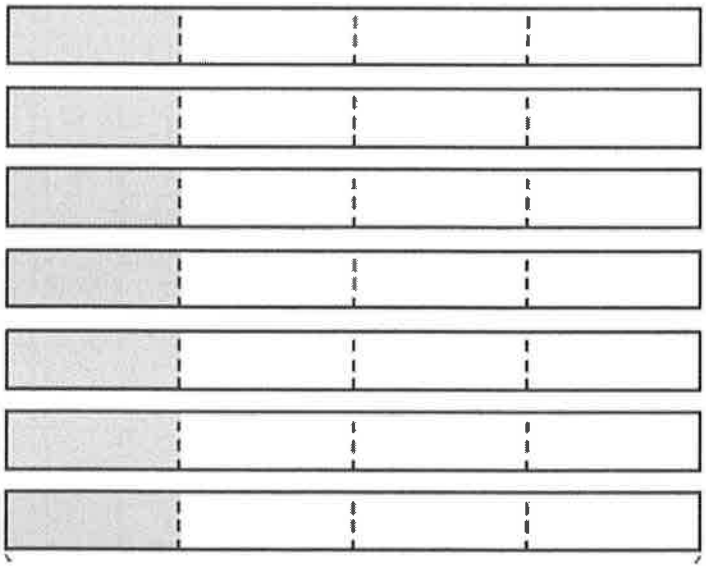
b. No, this is 4 times as long as the piece of rope.

c. Yes, because 12 feet of the rope gives 4 equal groups of 3 and then there are 3 feet left and dividing that into 4 equal parts is $\frac{3}{4}$ more feet.

6. Find the value of each expression.

$$\frac{1}{4} \times 7.$$

Solution : $\frac{7}{4}$

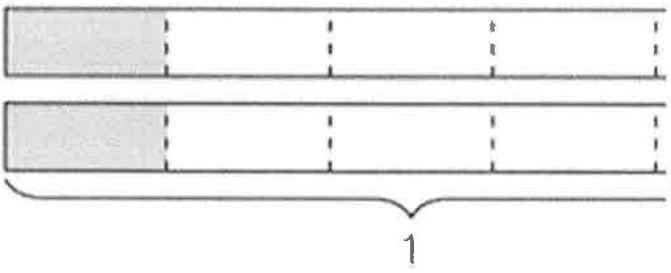


IM Fifth Grade Unit 2 Sample Questions

7. Kiran ran $\frac{1}{5}$ the length of his road, which is 9 miles long. How far did Kiran run? Show or explain your thinking.

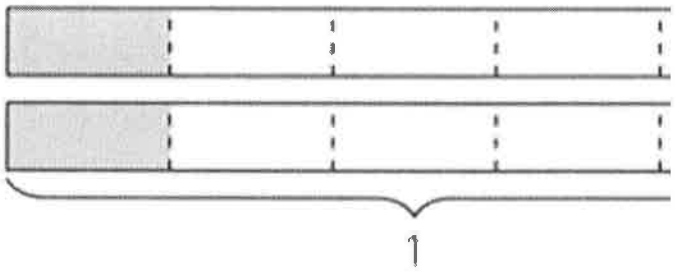
Solution : $\frac{9}{5}$ mile or $1\frac{4}{5}$ mile. Sample response: $9 \div 5 = \frac{9}{5}$

8. Explain how the diagram shows the expression: $2 \div 4$.



Solution :There are 2 whole rectangles and 1 out of 4 equal parts of the rectangles is shaded so that's $2 \div 4$.

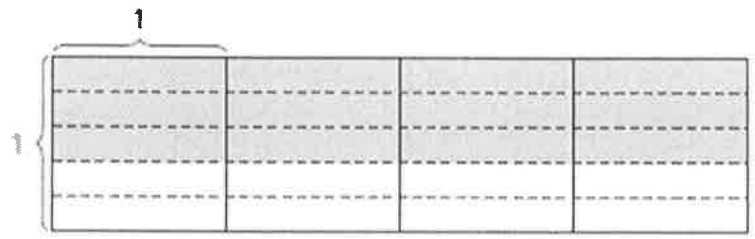
9. Explain how the diagram shows the expression $2 \times \frac{1}{4}$



Solution :There are 2 shaded parts and each shaded part is $\frac{1}{4}$ of a whole rectangle so that's $2 \times \frac{1}{4}$.

IM Fifth Grade Unit 2 Sample Questions

10. What is the area of the shaded region? Explain or show your reasoning.



Solution :

$\frac{12}{5}$ square units since there are 3×4 rectangles shaded and each is $\frac{1}{5}$ of a square unit.

11. Select all of the expressions that represent the shaded area in square feet.

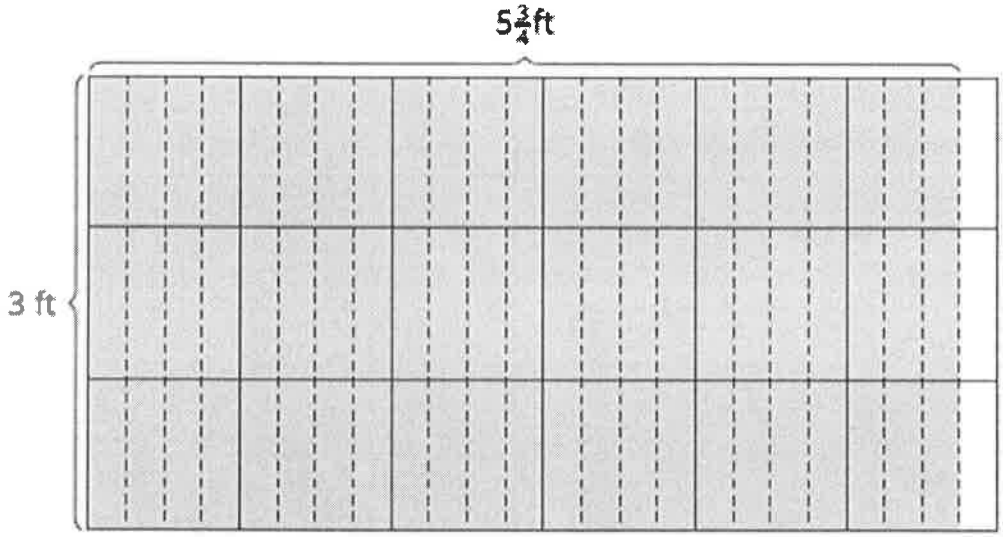
A. $3 + 5\frac{3}{4}$

B. $3 \times 5\frac{3}{4}$

C. $3 \times \left(5 + \frac{3}{4}\right)$

D. $(3 \times 5) + \frac{3}{4}$

E. $3 \times 6 - \left(3 \times \frac{1}{4}\right)$



Solution :

B, C, E

Sample expression: $3 \times 5 + 3 \times \frac{3}{4}$

IM Fifth Grade Unit 2 Sample Questions

12. Tyler says that $9\frac{11}{12} \times 5$ is a little less than 50.

    a. Do you agree with Tyler? Explain or show your reasoning.

    b. What is the value of $9\frac{11}{12} \times 5$?

Solution :

a. Yes, 10×5 is 50 and $9\frac{11}{12}$ is close to but less than 10 so $9\frac{11}{12} \times 5$ is a little less than 50.

b. $49\frac{7}{12}$, I took 10×5 and then subtracted $\frac{1}{12} \times 5$.

13. A banner at a sporting event is 12 feet long and $9\frac{2}{3}$ feet wide.

a. Find the area of the banner.

Solution :

$$12 \times 9\tfrac{2}{3} = 12 \times \left(9 + \tfrac{2}{3}\right)$$

$$= 12 \times 9 + \left(12 \times \tfrac{2}{3}\right) = 108 + 8 = 116$$

IM Fifth Grade Unit 2 Sample Questions

14. The distance from Lin's house to the store is 7 miles long. Lin drove $\frac{1}{4}$ of the distance and then stopped for gas. Lin's son drove $\frac{1}{3}$ of the distance and then stopped for picking up the grocery.

a. How many miles did Lin drive before stopping for gas? Explain or show your reasoning.
b. How many miles did Lin's son drive before stopping for picking up the grocery? Explain or show your reasoning.

Solution :

a. ¼ of the distance is ¼ of 7, which is the same as ¼ x 7 or 7÷ 4= $\frac{7}{4}$ miles.

b. 1/3 of the distance is 1/3 of 7, which is the same as 1/3 x 7 or 7÷ 3= $\frac{7}{3}$ miles.