# Family Support Materials

Multiplying and Dividing Fractions

In this unit, students use area concepts to represent and solve problems involving the multiplication of two fractions, and generalize that when they multiply two fractions, they need to multiply the two numerators and the two denominators to find their product. They also reason about the relationship between multiplication and division to divide a whole number by a unit fraction and a unit fraction by a whole number.

Section A: Fraction Multiplication

In this section, students build on their knowledge of fraction multiplication developed in the previous unit by using area concepts to understand the multiplication of a fraction times a fraction. Students draw diagrams to represent the fractional area. For example, students learn that the diagrams below can represent the situation "Kiran eats macaroni and cheese from a pan that is $\frac{1}{3}$ full. He eats $\frac{1}{4}$ of the remaining macaroni and cheese in the pan. How much of the whole pan did Kiran eat?"

| pan with $\frac{1}{3}$ left | eat $\frac{1}{4}$ of what's left | $\frac{1}{4}$ of $\frac{1}{3}$ is $\frac{1}{12}$ |
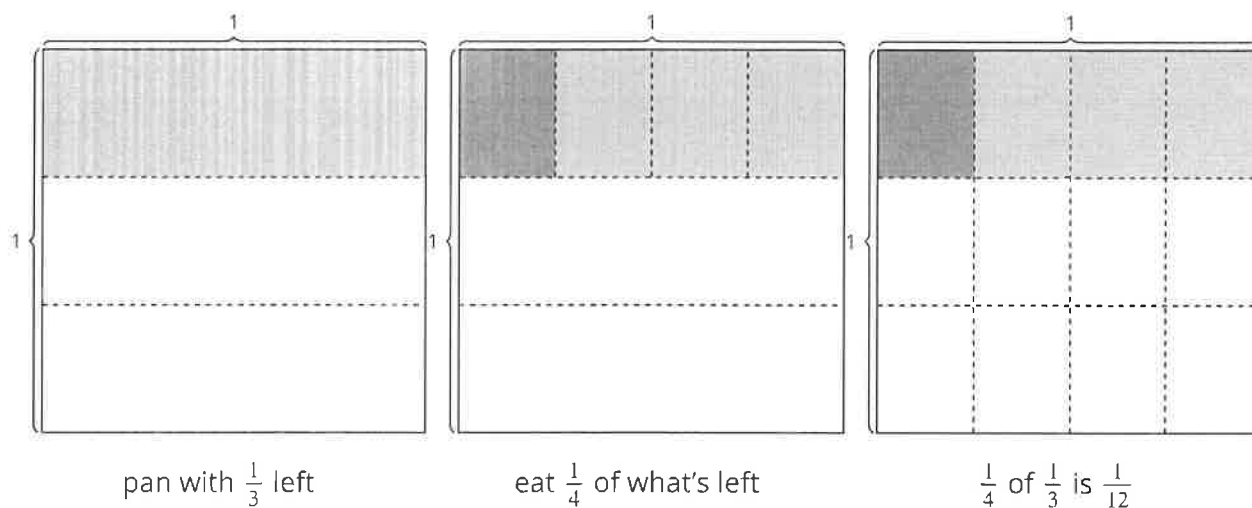
Students extend this conceptual understanding to multiply all types of fractions including fractions greater than 1 (for example, $\frac{7}{4}$). In each case, the students relate this multiplication to finding the area of a rectangle with fractions as side lengths. As the lessons progress, they notice that they can multiply the two numerators and the two denominators to find their product. This reasoning holds true for fractions greater than 1. For example, $\frac{3}{4} \times \frac{7}{5} = \frac{3 \times 7}{4 \times 5} = \frac{21}{20}$.

Section B: Fraction Division

The section begins by using whole numbers to recall that the size of the quotient depends, for example, on the amount being shared and the number of people sharing. That is, each student will get more pretzels if 3 students share 45 pretzels than if 3 students share 24 pretzels. Similarly, each student will get fewer pretzels if 6 students share 24 pretzels than if 3 students share 24 pretzels.

This thinking helps students understand why dividing a whole number by a unit fraction results in a quotient that is larger than the whole number. For example, $2 \div \frac{1}{3} = 6$ because there are 6 groups of $\frac{1}{3}$ in 2. As students draw diagrams and write expressions involving the division of unit fractions, students recognize the relationship between multiplication and division. For example, they may notice that $2 \div \frac{1}{3} = 6$ because $6 \times \frac{1}{3} = 2$, and that $\frac{1}{5} \div 2 = \frac{1}{10}$ is related to $2 \times \frac{1}{10} = \frac{1}{5}$.

Section C: Problem Solving with Fractions

In this section, students apply what they have learned in the previous sections through problem solving. Students see how fraction multiplication and division are useful in different contexts. They use the meaning of multiplication and division to decide which operation to use to solve various problems. As students share strategies, they may realize that some problems could be solved using either division or multiplication.

Try it at home!

Near the end of the unit, ask your student to solve the following question:

A painter was painting a wall yellow. He painted $\frac{1}{3}$ of the wall yellow before being told he needed to paint the wall blue. At the end of the day, he was able to cover up $\frac{1}{5}$ of the yellow wall in blue. How much of the entire wall is blue?

Questions that may be helpful as they work:

- Can you draw a diagram to help you solve the problem?
- What equation would you use to solve the problem?
- Can you solve this using division or multiplication instead?

Composite Number

A whole number with more than 1 factor pair.

6   6

1×6 2×3

6 is a composite number

Common denominator

The same denominator in two or more fractions. For instance, $\frac{1}{4}$ and $\frac{5}{4}$ have a common denominator.

Prime Number

A whole number that is greater than 1 and has exactly one factor pair: the number itself and 1.

13 has only two factors - itself and 1. So 13 is a prime number.

x

Mixed Number

A mixed number is a whole number, and a proper fraction represented together.

$1\frac{1}{4}$

Factor pair of a whole number

A pair of whole numbers that multiply to result in that number. For example, 5 and 4 are a factor pair of 20.

FACTOR PAIRS OF 8: 1×8 AND 2×4

Multiple of a number

The result of multiplying that number by a whole number. For example, a multiple of 3 is 15, because 15 is a result of multiplying 3 by 5.

Some multiples of 3

3   15   30

60   9

Equivalent fractions

Fractions that have the same size and describe the same point on the number line. For example, $\frac{1}{2}$ and $\frac{2}{4}$ are equivalent fractions.

denominator

The bottom part of a fraction that tells how many equal parts the whole was partitioned into.

$\frac{3}{5}$

numerator

The top part of a fraction that tells how many of the equal parts are being described.

$\frac{3}{5}$

Unit fraction

A fraction with 1 in the numerator.

$\frac{1}{2}$ $\frac{1}{4}$ $\frac{1}{23}$ $\frac{1}{5}$ $\frac{1}{6}$ $\frac{1}{8}$

quotient

The result in a division equation.

$4 = 8 \div 2$

$1 \div 2 = \frac{1}{2} = 1/2$

Divisor
the number we are dividing
by which can represent
the size of the groups
or the number of groups.

Dividend
a number to be
divided by another
number.

Equation
A statement that includes an equal sign (=).
It tells us that what is on one side of the sign
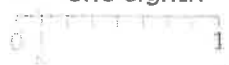is equal to what is on the other side.

$0 = 0$ $18 / 2 = 9$ $36 = 4 \times 9$

Expression
An expression has at least 2 numbers and at
least one math operation (such as addition,
subtraction, multiplication and division).

$3 \div 5$ $18 / 2$ $123 + 7$

fraction
A number used to describe
the parts of a whole that
has been partitioned
into equal parts.

$\frac{1}{8}$
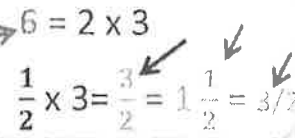
one eighth

product
The result of
multiplying some
numbers.

$6 = 2 \times 3$
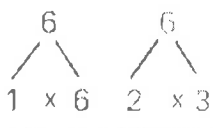
$\frac{1}{2} \times 3 = \frac{3}{2} = 1\frac{1}{2} = 3/2$

fù hé shù
复合数
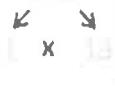
1×6 2×3

gōng fēn mǔ
公分母

zhì shù
质数
只有两个正
因数（1和自
己）的整数。

dài fēn shù
带分数

$1\frac{3}{4}$

yī duì yīn shù
一对因数

8 的因数: 1×8 和 2×4

bèi shù
倍数

3的倍数

3   15   30
60   9

děng zhí fēn shù
等值分数

shù zì xiàn
数字线

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fēn mǔ
分母

把单位 1
bǎ dān wèi
平均分成
píng jūn fēn chéng
若干部分
ruò gān bù fēn

$\dfrac{3}{⑤}$

fēn zǐ
分子

$\longrightarrow \dfrac{③}{5}$

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dān wèi fēn shù
单位分数

$①\frac{1}{4}$ $\frac{1}{①23}$ $①\frac{1}{8}$

shāng
商

$\rightarrow 4 = 8 \div 2$

$1 \div 2 = \dfrac{1}{2} = 1/2$

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chú shù
除 数

$\frac{3}{2} \div \frac{1}{2} = ?$

bèi chú shù
被除数

$\frac{3}{2} \div \frac{1}{2} = ?$

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fēn shù
分数

八分之一 $\dfrac{1}{8}$

one eighth

0 —————— 1

jī
积

$\rightarrow 6 = 2 \times 3$

$\dfrac{1}{2} \times 3 = \dfrac{3}{2} = 1\dfrac{1}{2} = 3/2$

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děng shì
等式

用等号 (=) 把数字和算
式连接起来，等号 (=)
的两边相等。

0 = 0   18 / 2 = 9   36 = 4 x 9

suàn shì
算式

用加号 (+) 减号 (-)，
乘号 (x)，除号 (÷) 等
符号连接数字的式子。

3 ÷ 5   18 / 2   123 + 7

3

IM Fifth Grade Unit 3 Sample Questions

1 a. Shade $\frac{1}{2}$ of $\frac{1}{5}$ of the square.

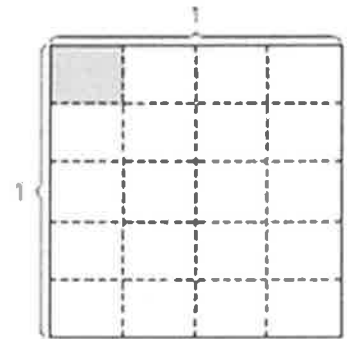b. Explain where you see $\frac{1}{2}$ of $\frac{1}{5}$ in your drawing.

Solution :
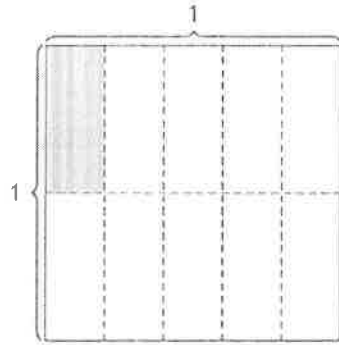
b. First I divided the squares into fifths vertically and then I shaded $\frac{1}{2}$ of one of those pieces.

2 a. Write an expression for how much of the square is shaded.

b. Find the value of your expression.

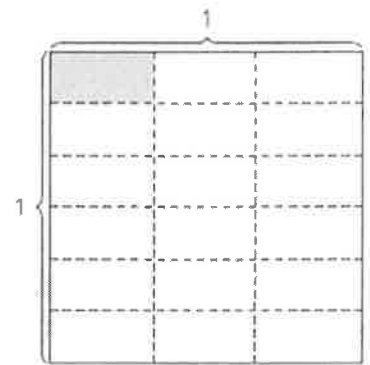a. $\frac{1}{4} \times \frac{1}{5}$ or $\frac{1}{5} \times \frac{1}{4}$

b. $\frac{1}{20}$

3. a. Write an equation representing the shaded part of the diagram.

b. Explain how the diagram shows each part of your equation.

a. $\frac{1}{3} \times \frac{1}{6} = \frac{1}{18}$ or $\frac{1}{6} \times \frac{1}{3} = \frac{1}{18}$
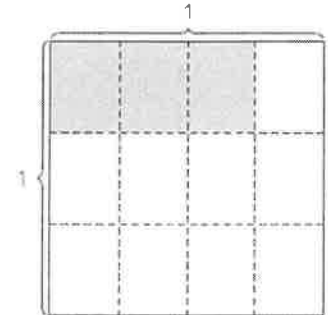
b. The square is divided in sixths horizontally so that's $\frac{1}{6}$. Then $\frac{1}{3}$ of one of those sixths is shaded. The value is $\frac{1}{18}$ because 1 of 18 rectangles is shaded.

IM Fifth Grade Unit 3 Sample Questions

4. a. Write an expression for the shaded region of the square.

   b. Explain how your expression matches the shaded region.
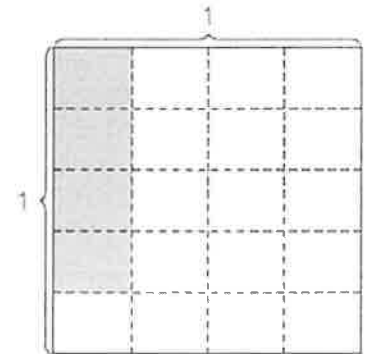
Solution :

a. $\frac{3}{4} \times \frac{1}{3}$

b. The top row is $\frac{1}{3}$ of the square and $\frac{3}{4}$ of that $\frac{1}{3}$ is shaded.

5. a. Write an expression for the shaded region of the square.

   b. Explain how your expression matches the shaded region.

Solution : a. $\frac{4}{5} \times \frac{1}{4}$ or $\frac{1}{4} \times \frac{4}{5}$

b. The first column is $\frac{1}{2}$ of the square and $\frac{4}{5}$ of that is shaded.

6 a. Write a multiplication expression for the area of the shaded region. Explain your reasoning.

b. What is the area of the shaded region in square units?
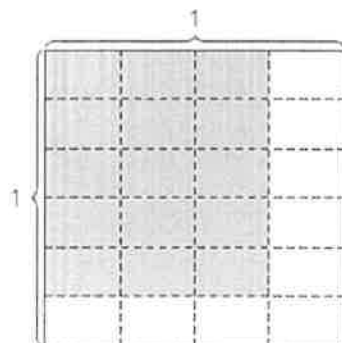
IM Fifth Grade Unit 3 Sample Questions

6. Solution :   a. $\frac{5}{6} \times \frac{3}{4}$ or $\frac{3}{4} \times \frac{5}{6}$.

The shaded region is $\frac{5}{6}$ of $\frac{3}{4}$ of the whole square.

b. $\frac{15}{24}$ or equivalent

7. Find the value that makes each equation true.

a. $\frac{7}{10} \times \frac{3}{5} = $ _____

b. $\frac{2}{5} \times$ _____ $= \frac{8}{45}$

c. _____ $\times \frac{4}{9} = \frac{28}{45}$

Solution :   a. $\frac{21}{50}$

b. $\frac{4}{9}$

c. $\frac{7}{5}$

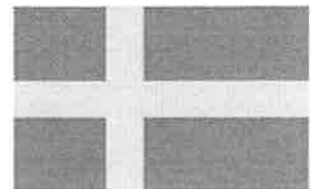8. This flag of Sweden is $3\frac{1}{5}$ inches wide and 2 inches tall.

The rectangle in the upper right is $\frac{9}{5}$ inches wide and $\frac{4}{5}$ inch tall.

a. What is the area of the whole flag?

b. What is the area of the rectangle in the upper right?

a. $6\frac{2}{5}$ square inches

b. $\frac{36}{25}$ square inches

IM Fifth Grade Unit 3 Sample Questions

9. For each pair of expressions, decide which is greater. Explain your choice without calculating the value of the expressions.
Solution :
a. $210 \div 3$ is greater because there are more groups of 3 in 210 than there are groups of 5.
b. $210 \div 3$ is greater because there are more groups of 3 in 210 than there are in 75.

10. A pan of macaroni and cheese is $\frac{1}{3}$ full. Four friends split the remaining macaroni and cheese
equally.
a. Make a drawing that represents the situation.
b. Write a division expression representing how much of a pan each friend gets.
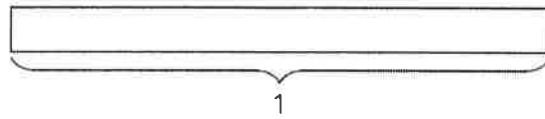c. Explain how the drawing shows the division expression.

Solution :
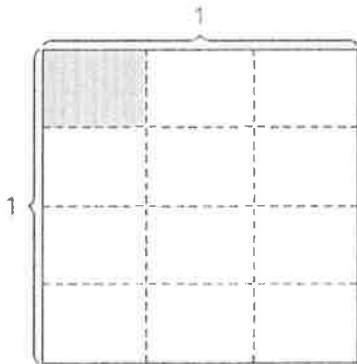
b. $\frac{1}{3} \div 4$

c. The left column is $\frac{1}{3}$ of the pan and I divided that into 4 equal pieces and shaded one of them.
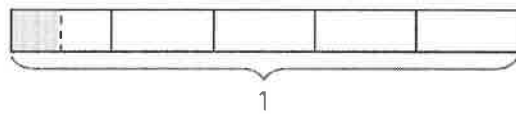
IM Fifth Grade Unit 3Sample Questions

11.   a. Use the diagram to represent the expression $\frac{1}{5} \div 2$.

1

b. Explain how the diagram shows $\frac{1}{5} \div 2$.

c. What is the value of $\frac{1}{5} \div 2$?

Solution :

1

b. I first divided the tape into 5 equal parts and then I divided one of those pieces, which is $\frac{1}{5}$, into two equal parts and shaded one of them.

c. $\frac{1}{10}$

12.

Mai has a strip of paper that is 3 feet long. She cuts it into $\frac{1}{4}$ foot strips.

a.   How many $\frac{1}{4}$ foot strips does Mai make? Explain or show your reasoning.

b. Write a division equation to represent your answer.

Solution :   a. 12 because there are 12 $\frac{1}{4}$s in 3.

b. $3 \div \frac{1}{4} = 12$.

IM Fifth Grade Unit 3 Sample Questions

13. Find the value of each expression.

a. $5 \div \frac{1}{4}$

b. $6 \div \frac{1}{4}$

c. $3 \div \frac{1}{6}$

d. $3 \div \frac{1}{7}$

Solution :

a. 20

b. 24

c. 18

d. 21

14. Solve each problem. Write an equation showing your answer.

a. There are 4 liters of water. How many $\frac{1}{2}$ liter bottles of water is that?

b. 4 friends split $\frac{1}{2}$ pound of dried fruit equally. How many pounds of fruit does each friend get?

Solution:

a. 8. $4 \div \frac{1}{2} = 8$

b. $\frac{1}{8} \cdot \frac{1}{2} \div 4 = \frac{1}{8}$

15. Find the value of each expression. Explain or show your reasoning.

a. $3 \div \frac{1}{4}$

b. $\frac{1}{5} \div 8$

a. 12 because there are $4 \frac{1}{4}$ s in 1 and so there are 3x4 or $12 \frac{1}{4}$ s in 3.

b. $\frac{1}{40}$ because there are $5 \frac{1}{5}$ s in a whole and so if I divide each one into 8 equal pieces there are 40 of those smaller pieces in a whole.